

Syrian Arab Republic

Ministry of Education and Higher Education

Arab University for Science and Technology

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Second Lecture

Modeling & Simulation

Lecturer Introduction

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Agenda

- Random Experiment.
- Sample Space (S).
- Event (E).
- Probability (P).
- Conditional Probability.
- Random Variable.
- Cumulative Distribution.
- Mean and Variance.
- Continuous Random Variable.
- Probability Density Function.
- Cumulative Distribution Function.

1- Random Experiment:

An experiment with a finite set of known outcomes, but whose result cannot be determined before the experiment runs; for example, rolling a die or flipping a coin.



The roll of a dice



The toss of (flipping) a coin

2- Sample Space (S):

It is the set of all possible outcomes of a random experiment and can be of two types:

- ❑ **Discrete:** consists of a finite number of outcomes, as with a die.
- ❑ **Continuous:** outcomes extend over an interval, such as the time required for a camera flash.



Flipping a coin

$$S = \{Head, Tail\}$$
$$S = \{H, T\}$$



$$S = \{x \mid 1.5 < x < 5\}$$

3- Event (E):

It is an event of a random experiment whose outcomes form a subset of the sample space, as shown in the figure.



The roll of a dice

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{2, 4, 6\}$$

4- Probability (P):

Probability is used to measure how likely the outcome of a random experiment is, and it is bounded between $[0, 1]$, where '0' indicates the event does not occur, while '1' indicates the event is certain to occur.

When computing the probability of an event, we distinguish two cases:

a) Equally likely outcomes:

$$P(E) = \frac{\text{\#of outcomes in event}(E)}{\text{Total \# of outcomes in sample space}(S)} = \frac{n(E)}{n(S)}$$

$$1 \geq P(E) \geq 0$$

Ex. $S = \{1, 2, 3, 4, 5, 6\}$

$$P(1) = \frac{1}{6}, \quad P(2) = \frac{1}{6}, \quad \dots, \quad P(6) = \frac{1}{6}$$

b) Non-equally likely outcomes:

the probability of an event equals the sum of the probabilities of its constituent outcomes, as in the example:

Random experiment: $\{a, b, c, d\}$ with probabilities $[0.1, 0.3, 0.5, 0.1]$, respectively. For the event $A = \{b, c\}$.

$$P(A) = 0.3 + 0.5 = 0.8.$$

5- Conditional Probability:

It is the probability of event B occurring given that event A has occurred, denoted as:

$$P(B \mid A)$$

It can be computed as follows (as shown in the figure):

$$P(B \mid A) = P(A \cap B) / P(A), \text{ provided } P(A) > 0.$$

$$\frac{P(A \cap B)}{P(A)} = \frac{\text{number of outcomes in } A \cap B}{\text{number of outcomes in } A}$$

6- Discrete Random Variable:

Is a function that assigns a real number to each outcome in the sample space of random experiment.

Denoted by an uppercase letter such as **X**.

7- Probability Mass Function:

For a discrete random variable X with possible values $x_1, x_2, \dots, x_n$, a **probability mass function** is a function such that

$$(1) \quad f(x_i) \geq 0$$

$$(2) \quad \sum_{i=1}^n f(x_i) = 1$$

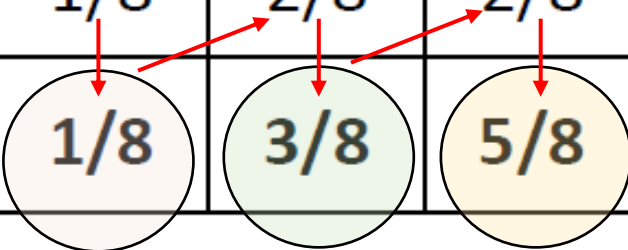
$$(3) \quad f(x_i) = P(X = x_i)$$

7- Cumulative Distribution:

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

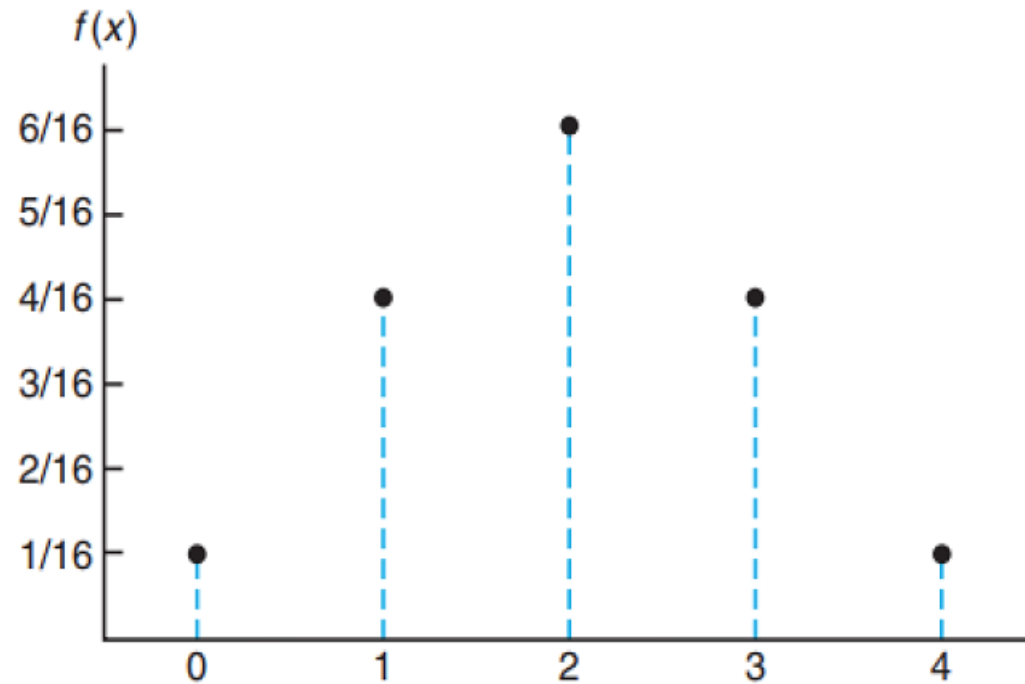
Example (1):

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8
$F(x) = P(X \leq x)$	1/8	3/8	5/8	7/8	8/8



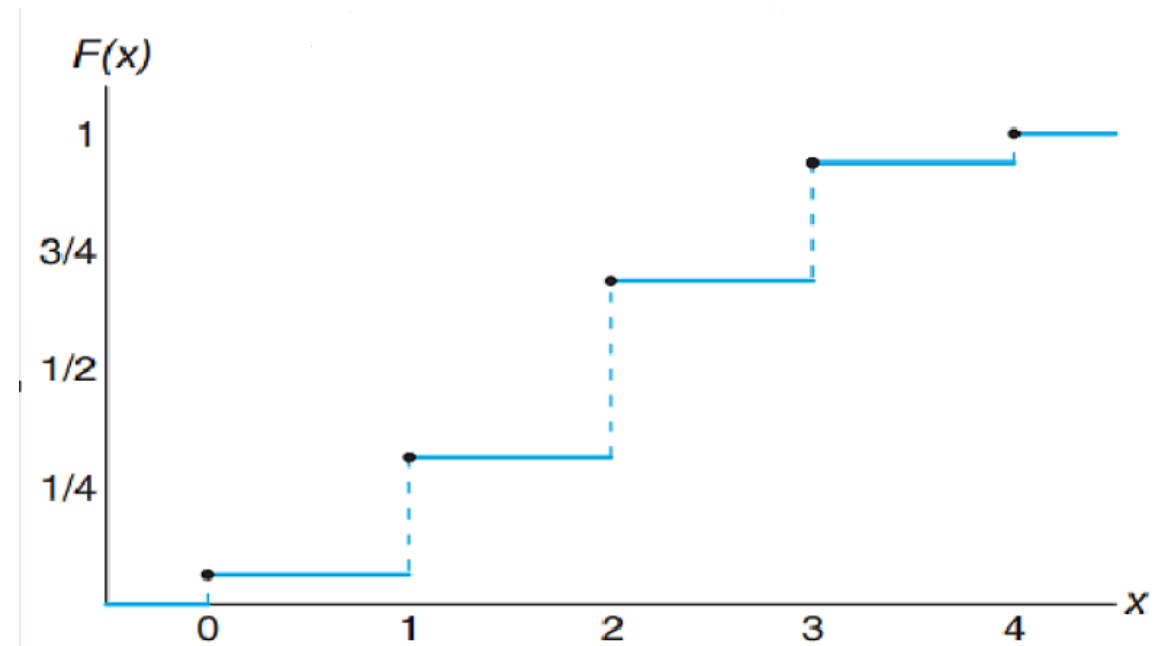
Example (2):

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16



Probability mass function plot.

x	0	1	2	3	4
$f(x) = P(X = x)$	1/16	4/16	6/16	4/16	1/16
$F(x) = P(X \leq x)$	1/16	5/16	11/16	15/16	16/16



Discrete cumulative distribution function.

8- Mean and Variance:

Mean: is a measure of the center or middle of the probability distribution.

Variance: is a measure of the dispersion.

The **mean** or **expected value** of the discrete random variable X , denoted as μ or $E(X)$, is

$$\mu = E(X) = \sum_x xf(x)$$

The **variance** of X , denoted as σ^2 or $V(X)$, is

$$\sigma^2 = V(X) = E(X - \mu)^2 = \sum_x (x - \mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2$$

The **standard deviation** of X is $\sigma = \sqrt{\sigma^2}$.

Example (1):

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

$$E(X) =$$

$$\sum x_i P(x_i) = (-2) \left(\frac{1}{8}\right) + (-1) \left(\frac{2}{8}\right) + (0) \left(\frac{2}{8}\right) + (1) \left(\frac{2}{8}\right) + (2) \left(\frac{1}{8}\right)$$
$$= 0$$

$$V(X) = E(X^2) - (E(X))^2$$

$$E(X) = 0$$

$$E(X^2)$$

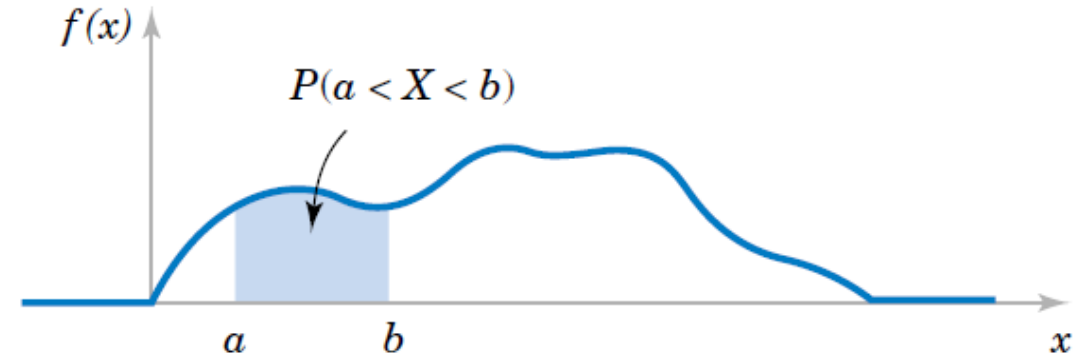
$$= \sum x_i^2 P(x_i) = (4) \left(\frac{1}{8}\right) + (1) \left(\frac{2}{8}\right) + (0) \left(\frac{2}{8}\right) + (1) \left(\frac{2}{8}\right) + (4) \left(\frac{1}{8}\right) = 1.5$$

$$V(X) = 1.5 - (0)^2 = 1.5,$$

$$\text{Standard Deviation } (\sigma) = \sqrt{1.5}$$

9- Continuous Random Variable:

If the range space of the random variable X is an interval.



10- Probability Density Function:

For a continuous random variable X , a **probability density function** is a function such that

$$(1) \quad f(x) \geq 0$$

$$(2) \quad \int_{-\infty}^{\infty} f(x) dx = 1$$

$$(3) \quad P(a \leq X \leq b) = \int_a^b f(x) dx = \text{area under } f(x) \text{ from } a \text{ to } b$$

for any a and b

Example1:

Suppose that $f(x) = e^{-x}$ for $x > 0$

Check the probability density function, then determine the
following probabilities:

1. $P(X < 1)$
2. $P(1 \leq X < 2.5)$
3. $P(X = 3)$
4. $P(X \geq 3)$

Check the probability density function:

$$\int_0^{\infty} e^{-x} dx$$

$$\int_0^{\infty} e^{-x} dx = -e^{-x} \Big|_0^{\infty} = (-e^{-\infty}) - (-e^0) = 0 + 1 = 1$$

$$1) P(X < 1)$$

$$\begin{aligned} P(X < 1) &= \int_0^1 e^{-x} dx = -e^{-x} \Big|_0^1 = (-e^{-1}) - (-e^0) \\ &= -0.367879 + 1 = 0.632121 \end{aligned}$$

$$2) P(1 \leq X < 2.5)$$

$$\begin{aligned} P(1 \leq X < 2.5) &= \int_1^{2.5} e^{-x} dx = -e^{-x} \Big|_1^{2.5} \\ &= (-e^{-2.5}) - (-e^{-1}) = -0.082085 + 0.367879 \\ &= 0.449964 \end{aligned}$$

$$3) P(X = 3)$$

$$P(X = 3) = 0$$

$$4) P(X \geq 3)$$

$$\begin{aligned} P(X \geq 3) &= \int_3^{\infty} e^{-x} dx = -e^{-x} \Big|_3^{\infty} = (-e^{-\infty}) - (-e^{-3}) \\ &= 0 + 0.049787 = 0.049787 \end{aligned}$$

11- Cumulative Distribution Function:

The **cumulative distribution function** of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(u) du$$

for $-\infty < x < \infty$.

Example1:

Find the cumulative distribution function $F(x)$ and use it to evaluate $P(0 < X \leq 1)$.

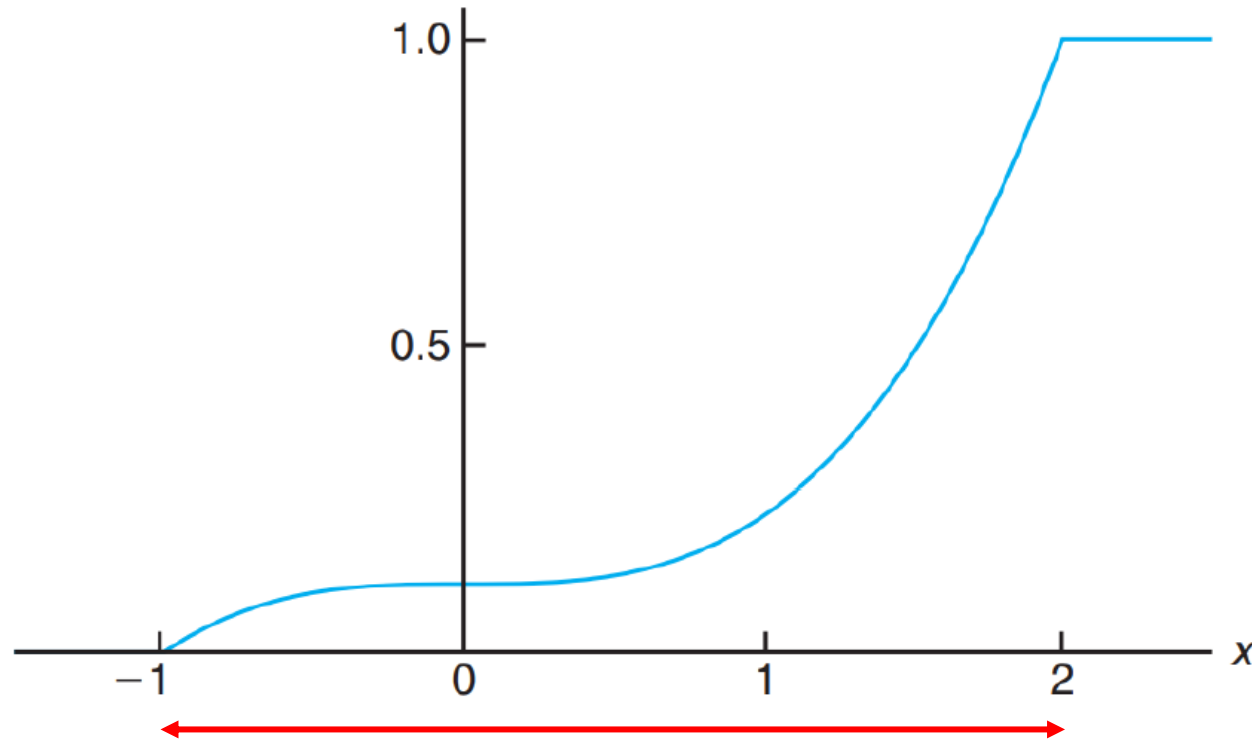
$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the cumulative distribution function $F(x)$

$$F(x) = \int_{-\infty}^x \frac{u^2}{3} du = \int_{-1}^x \frac{u^2}{3} du \quad \frac{x^2}{3}, \quad -1 < x < 2,$$

$$F(x) = \int_{-\infty}^x \frac{u^2}{3} du = \int_{-1}^x \frac{u^2}{3} du = \frac{u^3}{9} \Big|_{-1}^x = \left(\frac{(x)^3}{9} \right) - \left(\frac{(-1)^3}{9} \right) \\ = \frac{x^3}{9} + \frac{1}{9} = \frac{x^3 + 1}{9}$$

Find the cumulative distribution function $F(x)$



$$P(0 < X \leq 1) = F(1) - F(0) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

Thank you for listening